

Elements of analysis

B.01 Curve discussion

$$\begin{array}{lll} a) f(x) = x^3 - 9x; & d) f(x) = -0,5x^3 + 3,75x^2 - 6x; & g) f(x) = x^3 - 5x^2 - 8x + 12; \\ b) f(x) = -x^3 + 4x; & e) f(x) = \frac{3}{2}x^3 - 13,5x^2 + 22,5x; & h) f(x) = 2x^3 - 6x^2 + 8; \\ c) f(x) = \frac{1}{4}x^3 - \frac{25}{4}x; & f) f(x) = \frac{1}{40}x^3 + \frac{3}{20}x^2 - 0,9x; & i) f(x) = 2x^3 - 2x^2 - 2x + 2; \end{array}$$

B.02 Curve discussion

$$\begin{array}{lll} a) f(x) = (x-2)^2(x-3); & d) f(x) = -2(x-2)(x-1)(x-4); & g) f(x) = x(x^2 - 5x + 6); \\ b) f(x) = -2(x-1)^2(x-4); & e) f(x) = (x-2)(x-3)(x-4); & h) f(x) = x(-2x^2 + 10x - 8); \\ c) f(x) = -(x-0,5)^2(x+3); & f) f(x) = 0,5(x+3)(x-2)(x-1); & i) f(x) = x\left(\frac{3}{2}x^2 + 9x + 12\right); \end{array}$$

B.03 Curve discussion

$$\begin{array}{lll} a) f(x) = \frac{1}{4}x^4 - x^3 - 0,5x^2 + 3x + 1; & d) f(x) = \frac{1}{2}x^4 - \frac{8}{3}x^3 + x^2 + 12x; & g) f(x) = \frac{-\frac{1}{2}x^4 + \frac{1}{3}x^3 + x^2 - x - 1}{4}; \\ b) f(x) = 0,25x^4 - x^3 - 3x^2 + 8x + 2; & e) f(x) = -\frac{1}{2}x^4 - \frac{4}{3}x^3 + x^2 + 4x; & h) f(x) = \frac{\frac{3}{4}x^4 - x^3 - 6x^2 + 12x - 12}{6}; \\ c) f(x) = \frac{3}{4}x^4 - 10,5x^2 + 18x - 2; & f) f(x) = -\frac{3}{2}x^4 - x^3 + 3x^2 + 3x; & i) f(x) = 2x^4 - \frac{4}{3}x^3 - 4x^2 + 4x + 3; \end{array}$$

B.04 Curve discussion

$$\begin{array}{lll} a) f(x) = x^4 - 5x^2 + 4; & b) f(x) = 0,5x^4 - 5x^2 + 4,5; & c) f(x) = -\frac{1}{4}x^4 + \frac{25}{4}x^2 - 36; \\ d) f(x) = x^4 - 5x^3 + 6x^2; & e) f(x) = -x^4 + 3x^3 + 10x^2; & f) f(x) = \frac{1}{2}x^4 + \frac{5}{4}x^3 - \frac{3}{4}x^2; \end{array}$$

solutions

B.01	B.01	A	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \end{pmatrix} \begin{pmatrix} -1,732 & 10,392 \\ 1,732 & -10,392 \\ 0,000 & 0,000 \\ 3,000 & 0,000 \\ -3,000 & 0,000 \end{pmatrix}$	B.01	D	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \end{pmatrix} \begin{pmatrix} 4,000 & 4,000 \\ 1,000 & -2,750 \\ 2,500 & 0,6250 \\ 2,314 & 0,000 \\ 5,186 & 0,000 \\ 0,000 & 0,000 \end{pmatrix}$	B.01	g	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} -0,667 & 14,815 \\ 4,000 & -36,000 \\ 1,667 & -10,593 \\ -2,000 & 0,000 \\ 1,000 & 0,000 \\ 6,000 & 0,000 \end{pmatrix}$	a) Ytan= -9,00 X + 0,00 Ynor= 0,11 X + 0,00					
	B.01	B	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \end{pmatrix} \begin{pmatrix} 1,155 & 3,079 \\ -1,155 & -3,079 \\ 0,000 & 0,000 \\ 2,000 & 0,000 \\ -2,000 & 0,000 \end{pmatrix}$	B.01	E	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} 1,000 & 10,500 \\ 5,000 & -37,500 \\ 3,000 & -13,500 \\ 6,791 & 0,000 \\ 2,209 & 0,000 \\ 0,000 & 0,000 \end{pmatrix}$	B.01	h	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} 0,000 & 8,000 \\ 2,000 & 0,000 \\ 1,000 & 4,000 \\ -1,000 & 0,000 \\ 2,000 & 0,000 \\ 2,000 & 0,000 \end{pmatrix}$	b) Ytan= 4,00 X + 0,00 Ynor= -0,25 X + 0,00					
	B.01	C	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \end{pmatrix} \begin{pmatrix} -2,887 & 12,028 \\ 2,887 & -12,028 \\ 0,000 & 0,000 \\ 5,000 & 0,000 \\ -5,000 & 0,000 \end{pmatrix}$	B.01	F	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} -6,000 & 5,4000 \\ 2,000 & -1,000 \\ -2,000 & 2,200 \\ 3,708 & 0,000 \\ -9,708 & 0,000 \\ 0,000 & 0,000 \end{pmatrix}$	B.01	i	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} -0,333 & 2,370 \\ 1,000 & 0,000 \\ 0,333 & 1,185 \\ 1,000 & 0,000 \\ -1,000 & 0,000 \\ 1,000 & 0,000 \end{pmatrix}$	c) Ytan= -6,25 X + 0,00 Ynor= 0,16 X + 0,00					
	B.02	a	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} 2,000 & 0,000 \\ 2,667 & -0,148 \\ 2,333 & -0,074 \\ 3,000 & 0,000 \\ 2,000 & 0,000 \\ 2,000 & 0,000 \end{pmatrix}$	B.02	d	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} 3,215 & 4,225 \\ 1,451 & -1,262 \\ 2,333 & 1,481 \\ 2,000 & 0,000 \\ 1,000 & 0,000 \\ 4,000 & 0,000 \end{pmatrix}$	B.02	g	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} 0,7847 & 2,1126 \\ 2,5486 & -0,6311 \\ 1,6667 & 0,7407 \\ 3,000 & 0,000 \\ 2,000 & 0,000 \\ 0,000 & 0,000 \end{pmatrix}$	d) Ytan= 3,38 X - 7,81 Ynor= -0,30 X + 1,37					
	B.02	b	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} 3,000 & 8,000 \\ 1,000 & 0,000 \\ 2,000 & 4,000 \\ 4,000 & 0,000 \\ 1,000 & 0,000 \\ 1,000 & 0,000 \end{pmatrix}$	B.02	e	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} 2,423 & 0,385 \\ 3,577 & -0,385 \\ 3,000 & 0,000 \\ 2,000 & 0,000 \\ 3,000 & 0,000 \\ 4,000 & 0,000 \end{pmatrix}$	B.02	h	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} 2,8685 & 12,1292 \\ 0,4648 & -1,7588 \\ 1,6667 & 5,1852 \\ 1,000 & 0,000 \\ 4,000 & 0,000 \\ 0,000 & 0,000 \end{pmatrix}$	e) Ytan= -18,00 X + 40,50 Ynor= 0,0556 X - 13,67					
	B.02	c	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} 0,500 & 0,000 \\ -1,833 & -6,352 \\ -0,667 & -3,176 \\ -3,000 & 0,000 \\ 0,500 & 0,000 \\ 0,500 & 0,000 \end{pmatrix}$	B.02	f	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} -1,528 & 6,564 \\ 1,528 & -0,564 \\ 0,000 & 3,000 \\ 1,000 & 0,000 \\ 2,000 & 0,000 \\ -3,000 & 0,000 \end{pmatrix}$	B.02	i	$\begin{pmatrix} X_{E2} \\ X_{E2} \\ X_{WP} \\ NS_1 \\ NS_2 \\ NS_3 \end{pmatrix} \begin{pmatrix} -3,1547 & 4,6188 \\ -0,8453 & -4,6188 \\ -2,000 & 0,000 \\ -2,000 & 0,000 \\ -4,000 & 0,000 \\ 0,000 & 0,000 \end{pmatrix}$	f) Ytan= -1,20 X - 0,20 Ynor= 0,8333 X + 3,87					
	B.03	a	TP 3,00 -1,25 TP -1,00 -1,25 HP 1,00 2,75 WP1 2,15 0,528 WP2 -0,15 0,528	B.03	d	HP 2,00 14,67 TP -1,00 -7,83 TP 3,00 13,50 WP1 2,54 14,05 WP2 0,13 1,59	B.03	g	TP 0,50 -4,06 HP -1,00 -3,71 HP 1,00 -4,04 WP1 0,77 -4,05 WP2 -0,43 -3,86	B.04	a	HP 0,00 4,00 NS1 2,00 0,00 TP 1,58 -2,25 NS2 -2,00 0,00 TP -1,58 -2,25 NS3 1,00 0,00 WP 1 0,91 0,53 NS4 -1,00 0,00 WP 2 -0,91 0,53	B.04	d	TP 0,00 0,00 NS1 0,00 0,00 TP 2,59 -1,62 NS2 3,00 0,00 HP 1,16 2,08 NS3 2,00 0,00 WP 1 2,00 0,00 WP 2 0,50 0,94
	B.03	b	TP 4,00 -14,00 TP -2,00 -14,00 HP 1,00 6,25 WP1 2,73 -5,00 WP2 -0,73 -5,00	B.03	e	TP -1,00 -2,17 HP -2,00 -1,33 HP 1,00 3,17 WP1 0,22 0,89 WP2 -1,55 -1,72	B.03	h	TP 2,00 -1,33 TP -2,00 -6,67 HP 1,00 -1,04 WP1 1,54 -1,20 WP2 -0,87 -4,31	B.04	b	HP 0,00 4,50 NS1 3,00 0,00 TP 1,49 -4,14 NS2 -3,00 0,00 TP -1,49 -4,14 NS3 1,00 0,00 WP 1 1,29 -2,44 NS4 -1,00 0,00 WP 2 -1,29 -2,44	B.04	e	TP 0,00 0,00 NS1 0,00 0,00 HP -1,38 7,53 NS2 -2,00 0,00 HP 3,63 101,63 NS3 5,00 0,00 WP 1 -0,74 3,99 WP 2 2,24 58,85
	B.03	c	TP 2,00 4,00 TP -3,00 -89,75 HP 1,00 6,25 WP1 1,53 5,08 WP2 -1,53 -49,91	B.03	f	TP -0,50 -0,72 HP -1,00 -0,50 HP 1,00 3,50 WP1 0,43 1,73 WP2 -0,77 -0,60	B.03	i	HP 0,50 3,96 TP -1,00 -1,67 TP 1,00 3,67 WP1 0,77 3,80 WP2 -0,43 0,69	B.04	c	TP 0,00 -36,00 NS1 3,00 0,00 HP 0,59 -33,86 NS2 -3,00 0,00 HP -0,59 -33,86 NS3 4,00 0,00 WP 1 2,04 -14,30 NS4 -4,00 0,00 WP 2 -2,04 -14,30	B.04	f	HP 0,00 0,00 NS1 0,00 0,00 TP 0,34 -0,03 NS2 0,50 0,00 TP -2,21 -5,23 NS3 -3,00 0,00 WP 1 0,18 -0,02 WP 2 -1,43 -3,08

function $f(x)$ is given

$$f(x) = (0,5 * X + 1) * e^{(-1 * X + 2)}$$

a) show that $f'(x) = (0,5 * X + 1) * e^{(-0,5 * X + 0)}$

b) show that $f''(x) = (0,5 * X + 1) * e^{(-0,25 * X - 0,5)}$

c) show that $f'''(x) = (0,5 * X + 1) * e^{(-0,125 * X - 0,5)}$

d) zero points:

2,0	0,0
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e) Extrem points Max

0,00	5,44
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 and $f'(X_e) = -1,36$

f) inflection point IP

-2	4,0
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 and $f''(X_{wp}) = -0,25$

function $f(x)$ is given

$$f(x) = (-1 * X + 1) * e^{(-2 * X^2 + 1 * X + 2)}$$

a) show that $f'(x) = (-1 * X + 1) * e^{(2 * X^2 - 5 * X - 1)}$

b) show that $f''(x) = (-1 * X + 1) * e^{(-2 * X^2 + 9 * X - 4)}$

c) show that $f'''(x) = (-1 * X + 1) * e^{(2 * X^2 - 13 * X + 13)}$

d) zero points:

-0,781	0
1,281	0

e) Extrem points

2,686	-1,805	Min
-0,186	5,712	HP

 NB: $f''(X_e) = 1,1$
NB: $f''(X_e) = -18,8$

f) inflection point

4	-1,294	IP1
0,5	3,297	IP2

 NB: $f'''(X_{ip}) = -0,3$
NB: $f'''(X_{ip}) = 11,5$

function $f(x)$ is given

$$f(x) = (2 \cdot x + 1) \cdot e^{(2 \cdot x - 1)}$$

a) show that $f'(x) = (4 \cdot x + 0) \cdot e^{(2 \cdot x + 1)}$

b) show that $f''(x) = (8 \cdot x + 4) \cdot e^{(2 \cdot x + 1)}$

c) show that $f'''(x) = (16,000 \cdot x + 16) \cdot e^{(2 \cdot x + 1)}$

d) zero points:

0,5	0,0
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e) Extrem points Min

0,00	-2,7
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 and $f'(X_e) = 10,87$

f) inflection point IP

-0,5	-2,0
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 and $f''(X_{wp}) = 8,00$

function $f(x)$ is given

$$f(x) = (-1 \cdot x^2 + 2 \cdot x + 3) \cdot e^{(-1 \cdot x + 2)}$$

a) show that $f'(x) = (1 \cdot x^2 - 4 \cdot x - 1) \cdot e^{(-1 \cdot x + 2)}$

b) show that $f''(x) = (-1 \cdot x^2 + 6 \cdot x - 3) \cdot e^{(-1 \cdot x + 2)}$

c) show that $f'''(x) = (1 \cdot x^2 - 8 \cdot x + 9) \cdot e^{(-1 \cdot x + 2)}$

d) zero points:

-1,00	0
3,00	0

e) Extrem points

4,24	-0,692	Min
-0,24	23,130	HP

 NB: $f''(X_e) = 0,5$
NB: $f''(X_e) = -41,8$

f) inflection point

5,45	-0,502	IP1
0,55	16,183	IP2

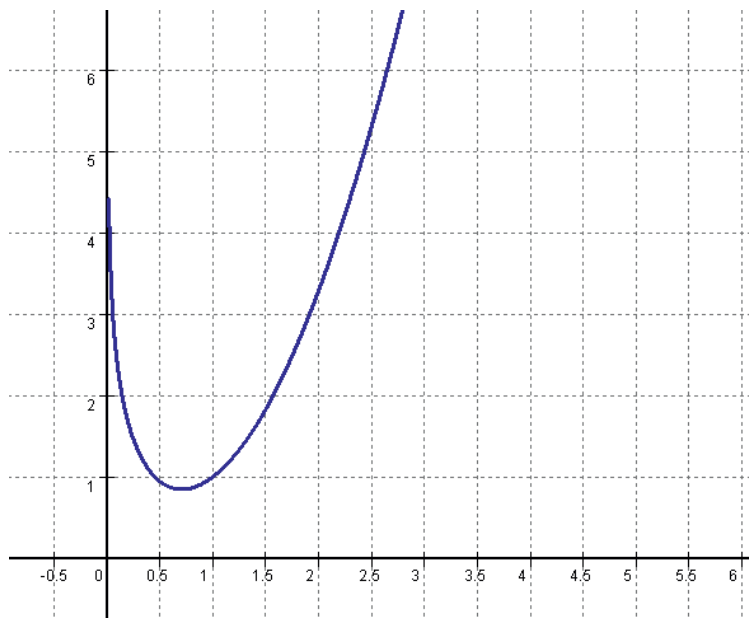
 NB: $f'''(X_{ip}) = -0,2$
NB: $f'''(X_{ip}) = 20,9$

$$f(x) = x^2 - \ln x$$

Zero : NO

Min : (0,707; 0,847)

IP : NO

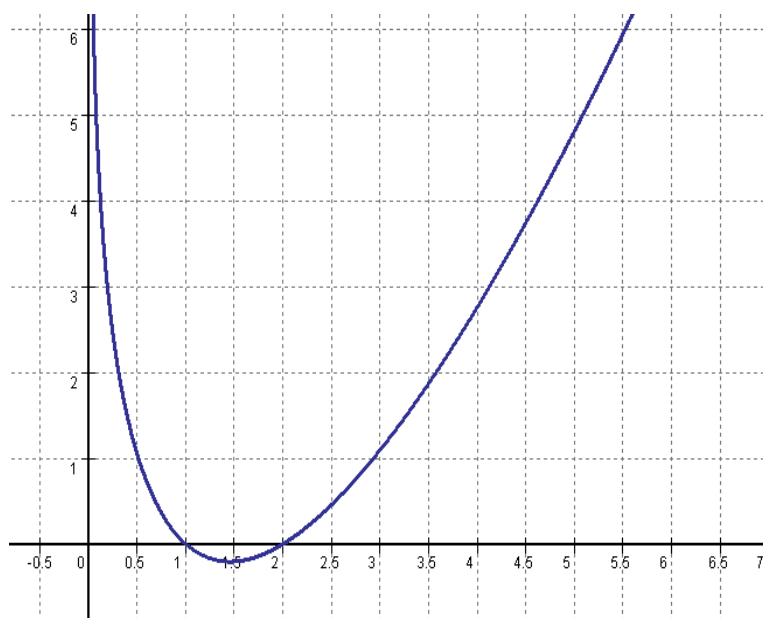


$$f(x) = (x - 2) \ln x$$

Zero : 1; 2

Min : (1,4547; -0,204)

IP : NO



$$f(x) = 3x \ln(x^{\frac{1}{3}})$$

Zero : 1

Min : (0,3679; -0,368)

IP : NO

