There is a version of the second partials test that can be used to determine what kind of constrained relative extremum corresponds to each critical point (a, b) of F. The techniques required to carry out such an analysis are discussed in more advanced texts, but in this text, we will assume that if f has a constrained maximum (minimum), it will be given by the largest (smallest) of the critical values $f(a, b)$. Here is a summary of the procedure used in the method of Lagrange multipliers.

A Procedure for Applying the Method of Lagrange Multipliers

Step 1. Write the problem in the form:

$$\text{Maximize (minimize) } f(x, y) \text{ subject to } g(x, y) = k$$

Step 2. Simultaneously solve the equations

$$f_x(x, y) = \lambda g_x(x, y)$$
$$f_y(x, y) = \lambda g_y(x, y)$$
$$g(x, y) = k$$

Step 3. Evaluate f at all points found in step 2. If the required maximum (minimum) exists, it will be the largest (smallest) of these values.

A geometric justification of the multiplier method is given at the end of this section. In the following example, the method is used to solve the problem from Example 5.1 in Section 5 of Chapter 3.

EXAMPLE 4.1

The highway department is planning to build a picnic area for motorists along a major highway. It is to be rectangular with an area of 5,000 square yards and is to be fenced off on the three sides not adjacent to the highway. What is the least amount of fencing that will be needed to complete the job?

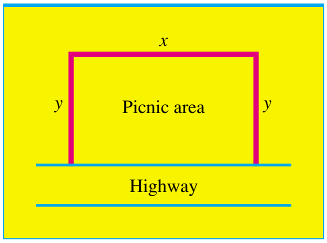

FIGURE 7.21 Rectangular picnic area.

Solution

Label the sides of the picnic area as indicated in Figure 7.21 and let f denote the amount of fencing required. Then,

$$f(x, y) = x + 2y$$

The goal is to minimize f subject to the constraint that the area must be 5,000 square yards; that is, subject to the constraint

$$xy = 5{,}000$$