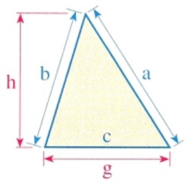
## Dreieck

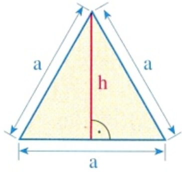



$$A = \frac{g \cdot h}{2}$$

$$A = \frac{c \cdot h_c}{2} = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2}$$

$$u = a + b + c$$

## Gleichseitiges Dreieck

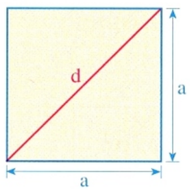



$$h = \frac{a}{2}\sqrt{3} \qquad r_U = \frac{a}{\sqrt{3}}$$

$$A = \frac{a^2}{4}\sqrt{3} \qquad r_I = \frac{\sqrt{3}a}{6} = \frac{1}{2} \cdot r_U$$

$$u = 3a$$

## Quadrat

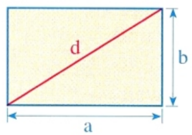



$$A = a \cdot a = a^2$$

$$u = 4a$$

$$d = a\sqrt{2}$$

## Rechteck

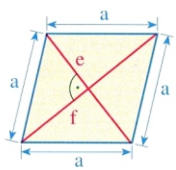



$$A = a \cdot b$$

$$u = 2a + 2b$$

$$d = \sqrt{a^2 + b^2}$$

## Raute

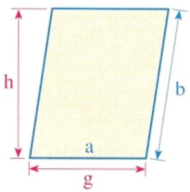



$$A = \frac{e \cdot f}{2}$$

$$u = 4a$$

e, f: Länge der Diagonalen

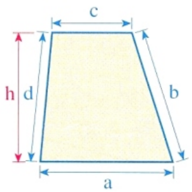
## Parallelogramm




$$A = g \cdot h$$

$$u = 2a + 2b$$

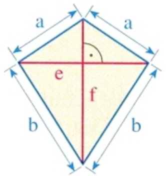
## Trapez




$$A = \frac{a + c}{2} \cdot h$$

$$u = a + b + c + d$$

## Drachenviereck

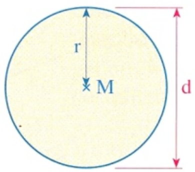



$$A = \frac{e \cdot f}{2}$$

$$u = 2a + 2b$$

e, f: Länge der Diagonalen

## Kreis



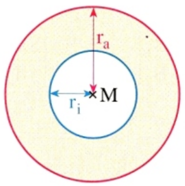

$$A = \pi \cdot r^2$$

$$A = \pi \cdot \frac{d^2}{4}$$

$$u = 2\pi \cdot r$$

$$u = \pi \cdot d$$

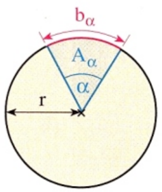
## Kreisring




$$A = \pi \cdot r_a^2 - \pi \cdot r_i^2$$

$$= \pi \cdot (r_a^2 - r_i^2)$$

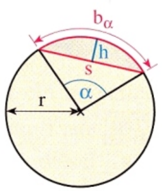
## Kreisausschnitt




$$b_\alpha = u \cdot \frac{\alpha}{360°} = 2\pi r \cdot \frac{\alpha}{360°}$$

$$A_\alpha = \pi \cdot r^2 \cdot \frac{\alpha}{360°} = \frac{b_\alpha \cdot r}{2}$$

## Kreisabschnitt

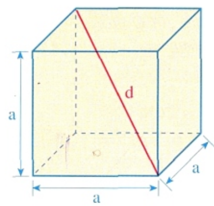



$$A = \frac{1}{2} b_\alpha \cdot r - \frac{1}{2} s(r - h)$$

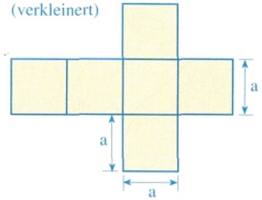
s: Länge der Kreissehne;

h: Höhe des Kreisabschnitts

## Würfel




(verkleinert)



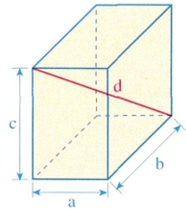

$$V = a^3$$

$$A_O = 6 \cdot a^2$$

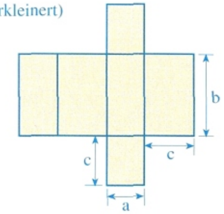
$$d = a\sqrt{3}$$

d: Länge der Raumdiagonalen

## Quader




(verkleinert)




$$V = a \cdot b \cdot c$$

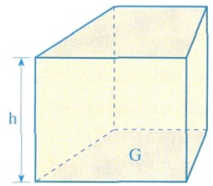
$$A_O = 2 \cdot (ab + bc + ca)$$

$$= 2ab + 2bc + 2ca$$

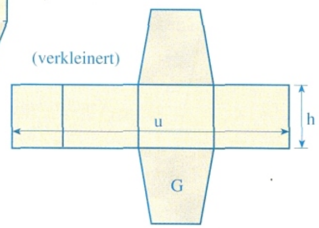
$$d = \sqrt{a^2 + b^2 + c^2}$$

d: Länge der Raumdiagonalen

## Prisma




(verkleinert)




$$V = A_G \cdot h$$

$$A_M = u \cdot h$$

$$A_O = 2 \cdot A_G + A_M = 2 \cdot A_G + u \cdot h$$

*Sonderfälle des Prismas:*

(1) Die Grundfläche ist ein Quadrat:

$$V = a^2 \cdot h$$

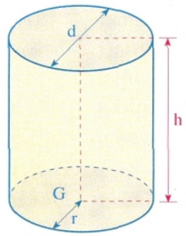
$$A_O = 2a^2 + 4a \cdot h$$

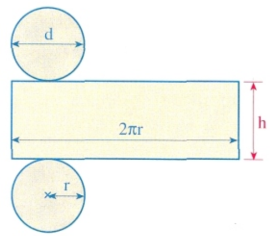
(2) Die Grundfläche ist ein gleichseitiges Dreieck:

$$V = \frac{1}{4} a^2 \sqrt{3} \cdot h$$

$$A_O = \frac{1}{2} a^2 \sqrt{3} + 3a \cdot h$$

## Zylinder




$$V = A_G \cdot h = \pi r^2 \cdot h = \frac{\pi d^2}{4} \cdot h$$

$$A_M = 2\pi r \cdot h = \pi d \cdot h$$

$$A_O = 2 \cdot A_G + A_M = 2\pi r^2 + 2\pi r \cdot h$$

$$= 2 \cdot \frac{\pi d^2}{4} + \pi d \cdot h$$

## Der Umkreisradius eines Dreiecks

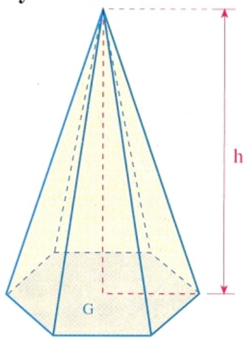
$$R = \frac{a}{2\sin\alpha} = \frac{b}{2\sin\beta} = \frac{c}{2\sin\gamma} = \frac{abc}{4A}$$

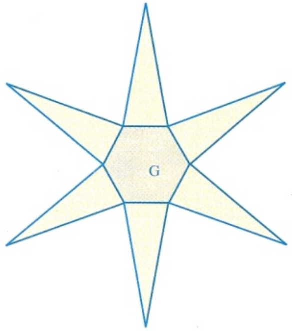
## Der Umkreisradius eines gleichschenkliges Dreiecks

$$R = (4h^2 + c^2)/(8h)$$

wobei C - Basis, h - Höhe

## Pyramide




$V = \frac{1}{3} \cdot A_G \cdot h$

$A_O = A_G + A_M$

*Quadratische Pyramide (Grundfläche ist ein Quadrat mit der Seitenlänge a):*

$V = \frac{1}{3} \cdot a^2 \cdot h$

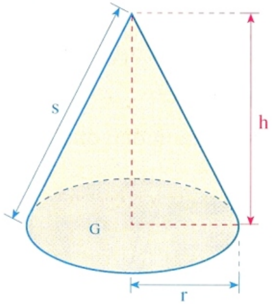
$A_M = 4 \cdot \frac{a \cdot h_s}{2} = 2ah_s$

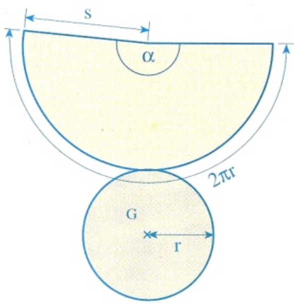
$A_O = a^2 + 2ah_s$

$h_s = \sqrt{h^2 + \left(\frac{a}{2}\right)^2}$

$h_s$: Höhe einer Seitenfläche

## Kegel




$V = \frac{1}{3} \cdot A_G \cdot h = \frac{1}{3} \cdot \pi r^2 \cdot h = \frac{\pi d^2}{12} \cdot h$

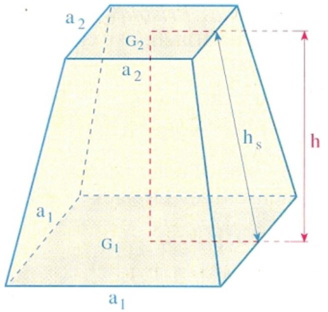
$A_M = \pi r s$

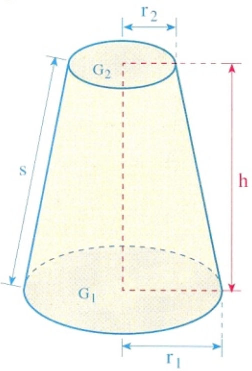
$A_O = A_G + A_M = \pi r^2 + \pi r s = \frac{\pi d^2}{4} + \frac{\pi d}{2} \cdot s$

$s = \sqrt{h^2 + r^2}$

$\alpha = \frac{360° \cdot r}{s}$

## Pyramidenstumpf und Kegelstumpf




$V = \frac{1}{3} h \left(A_{G_1} + \sqrt{A_{G_1} \cdot A_{G_2}} + A_{G_2}\right)$

*Quadratischer Pyramidenstumpf:*

$V = \frac{1}{3} h (a_1^2 + a_1 \cdot a_2 + a_2^2)$

$A_O = a_1^2 + a_2^2 + 4 \cdot \frac{a_1 + a_2}{2} \cdot h_s$

*Kegelstumpf:*

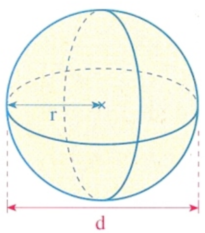
$V = \frac{1}{3} \pi h (r_1^2 + r_1 \cdot r_2 + r_2^2)$

$A_M = \pi s (r_1 + r_2)$

$A_O = \pi r_1^2 + \pi r_2^2 + \pi s (r_1 + r)$

$s = \sqrt{h^2 + (r_1 - r)^2}$

## Kugel




$V = \frac{4}{3} \pi r^3 = \frac{\pi d^3}{6}$

$A_O = 4 \pi r^2 = \pi d^2$

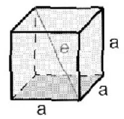
$$sin = \frac{GK}{H};\ \cos = \frac{AK}{H};\ \tan = \frac{GK}{AK};$$

## Körper mit ebenen Begrenzungsflächen

| | | | |
|---|---|---|---|
| Grundfläche $A_G$ | Deckfläche $A_D$ | Mantelfläche $A_M$ | Oberfläche $A_O$ |
| Kantenlängen a, b, c | Körperhöhe h | Körperdiagonale e | Seitenflächenhöhe $h_s$ |
| Seitenkante s | Volumen V | | |

### Prismen — $V = A_G h$ — $A_O = 2A_G + A_M$ — $A_G = A_D$

**Würfel**



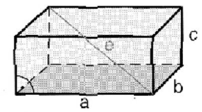

$e = a\sqrt{3}$

$A_M = 4a^2$

$A_O = 6a^2$

$V = a^3$

**Quader**



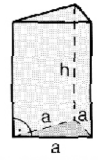

$e = \sqrt{a^2 + b^2 + c^2}$

$A_M = 2(ac + bc)$

$A_O = 2(ab + ac + bc)$

$V = abc$

**regelmäßiges dreiseitiges Prisma**

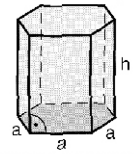



$A_M = 3ah$

$A_O = \frac{a}{2}(a\sqrt{3} + 6h)$

$V = \frac{a^2}{4} h\sqrt{3}$

**regelmäßiges sechsseitiges Prisma**



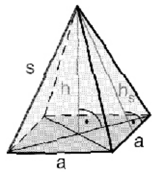

$A_M = 6ah$

$A_O = 3a(a\sqrt{3} + 2h)$

$V = \frac{3a^2}{2} h\sqrt{3}$

### Pyramiden — $V = \frac{1}{3} A_G h$ — $A_O = A_G + A_M$

**gerade quadratische Pyramide**

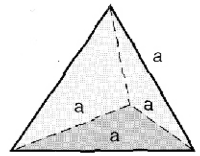



$A_M = 2ah_s$

$A_O = a(a + 2h_s)$

$V = \frac{1}{3} a^2 h$

**Tetraeder**

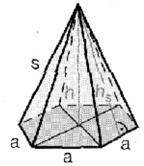



$A_M = \frac{3a^2}{4}\sqrt{3}$

$A_O = a^2\sqrt{3}$

$V = \frac{a^3}{12}\sqrt{2}$

**regelmäßige sechsseitige Pyramide**



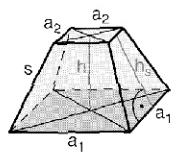

$A_M = 3ah_s$

$A_O = \frac{3}{2}a(a\sqrt{3} + 2h_s)$

$V = \frac{a^2}{2} h\sqrt{3}$

### Pyramidenstümpfe — $V = \frac{h}{3}(A_G + \sqrt{A_G A_D} + A_D)$ — $A_O = A_G + A_D + A_M$

**quadratischer Pyramidenstumpf**

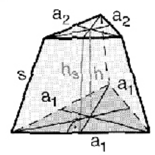



$A_M = 2(a_1 + a_2)h_s$

$A_O = a_1^2 + 2(a_1 + a_2)h_s + a_2^2$

$V = \frac{1}{3} h(a_1^2 + a_1 a_2 + a_2^2)$

**regelmäßiger dreiseitiger Pyramidenstumpf**

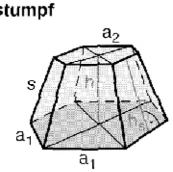



$A_M = \frac{3}{2}(a_1 + a_2)h_s$

$A_O = \frac{\sqrt{3}}{4}(a_1^2 + a_2^2) + \frac{3}{2}(a_1 + a_2)h_s$

$V = \frac{\sqrt{3}}{12} h(a_1^2 + a_1 a_2 + a_2^2)$

**regelmäßiger sechsseitiger Pyramidenstumpf**




$A_M = 3(a_1 + a_2)h_s$

$A_O = \frac{3\sqrt{3}}{2}(a_1^2 + a_2^2) + 3(a_1 + a_2)h_s$

$V = \frac{\sqrt{3}}{2} h(a_1^2 + a_1 a_2 + a_2^2)$